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We show that neutrino telescopes, optimized for detecting neutrinos of TeV to PeV energy, can reveal threshold effects associated with TeV-scale gravity. The signature is an increase with energy of the cross section beyond what is predicted by the Standard Model. The advantage of the method is that the neutrino cross section is measured in an energy region where i) the models are perturbatively calculable and ii) the Standard Model neutrino cross section can be reliably calculated so that any deviation can be conclusively identified.

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Motivated by the absence of a self-consistent theory of quantum gravity and the unresolved hierarchy problem between the electroweak scale (10^2 GeV) and the Planck scale (10^{19} GeV), a great deal of attention has been given to theories of low-scale quantum gravity which envision significant quantum gravity effects at an energy scale of the order of $M_s \sim 1$ TeV [1,2]. In these scenarios, potentially large effects on high energy processes may occur due to the contributions from, *e.g.*, Kaluza-Klein excitations of gravitons (KK) or other stringy states near M_s. This results in neutrino cross sections which gradually increase to the scale of quarks and leptons. Another motivation for models in which cross sections at TeV scale become enhanced is the ultra-high energy cosmic ray problem. Protons above the GZK cutoff ($\sim 10^{19}$ eV) interact with the cosmic microwave background cataclysmically by the Δ-resonance [3,4]. Thus, the cosmic ray events observed above this energy must be produced by local sources, or involve new physics. Local sources of particles of such energy being unlikely, many exotic solutions have been proposed [5,6]. A solution which has received a great deal of attention in recent literature proposes that neutrinos with enhanced cross sections at GZK energies constitute the highest energy cosmic rays. This solution requires neutrino-nucleon cross sections on the scale of 10's of mbarnes. The prospects for neutrinos acquiring such interactions have been studied [7–9]. Unfortunately, most scenarios of low-scale quantum gravity as low-energy effecive theories are valid only up to $\lesssim M_s$. Above this scale, the naive calculations typically violate unitarity. One has to introduce some ad hoc unitarization scheme, since the fundamental theory, such as a realistic string theory, is yet unavailable. It is also very difficult to reliably predict the parton distribution functions needed at GZK energies in neutrino-nucleon interactions. For these reasons, studies of ultra-high energy ($\sim 10^{20}$ eV) quantum gravity enhancements to neutrino-nucleon interactions are extremely speculative.

These problems are far more manageable at energies below or near M_s. Unitarity may not be violated at this scale, calculations are generally perturbative and the relevant parton distributions are known at these energies [10]. Therefore, the TeV scale provides a natural scale for probing the features of low-scale quantum gravity models. These tests include searches in colliders such as the Large Hadron Collider (LHC) and the Tevatron [11–13]. This letter discusses another class of experiments capable of testing features of low-scale quantum gravity: multi-TeV to PeV neutrino astrophysics.

We have recently witnessed first light of neutrino telescopes optimized to detect neutrinos in the TeV to PeV energy range [14,15]. This is the range of laboratory energies where the onset of TeV-scale gravity effects on the neutrino cross section will first manifest itself. For a neutrino flux Φ, the number of events N_ν observed in a neutrino detector of effective area A is given by the convolution over energy of the quantity $A \times \Phi \times n \times \sigma_\nu \times R_\mu$. Here n is the density of the target that converts a neutrino with cross section σ_ν into a detected muon of range R_μ. In our discussion the detected neutrino flux plays a secondary role. It may represent the atmospheric neutrino flux or the flux of neutrinos of hundreds of TeV anticipated from gamma ray bursts. The key observation is that, in the energy range relevant to the onset of TeV-scale gravity, the Earth becomes opaque to the passage of neutrinos. An increase with energy of the cross section for neutrinos to interact with matter beyond a level calculated in the Standard Model, will signal the onset of new physics including the increase anticipated as a consequence of TeV-scale gravity effects. It is important to recognize that the Standard Model cross section is computed from nucleon structure functions probed by HERA experiments in this energy range. The Standard Model baseline against which to measure new physics is known.

From an experimental point of view, the effects of new physics are rather dramatic. With increasing energy, the neutrinos are absorbed by the Earth and eventually, when the Earth becomes opaque, only events originating in the atmosphere near the horizon are observed. Near the horizon, the atmosphere represents a target density for converting neutrinos of $36\,\mathrm{kg/cm^2}$. We will discuss this in more detail below, but the sensitivity of such a measurement to the neutrino cross section can be simply understood as follows: measuring the number of events generated in the detector by the same flux Φ (i.e. measuring $A \times \Phi \times n \times \sigma_\nu \times R_\mu$) on two different targets (air and water/ice) contains information on the cross section. We here assume that, to first order, the range of the muon is understood because it is reliably calculated from QED (to second order, the range will also be modified by the new physics, e.g. by the contribution of deep inelastic scattering of the muon). These measurements, though challenging for existing instruments like AMANDA and the detectors in the Mediterranean, should not be a great challenge for a second-generation detectors such as IceCube. Unlike first-generation neutrino telescopes, IceCube can measure energy, and can therefore separate interesting high energy events from the large background of lower energy atmospheric neutrinos by energy measurement. The instrument can identify high energy neutrinos over 4π solid angle, and not just in the lower hemisphere where they are identified by their penetration of the Earth, as is the case with AMANDA.

For the sake of illustration, we will examine three classes of low-scale quantum gravity models:

1). ADD scenario [1]: Large extra dimensions ($R <$ 0.1 mm) with a flat Minkowski metric. The large effects are due to the high degeneracy of the light KK gravitons of mass $m_{KK} \sim 1/R$. The theory saturates perturbative unitarity at $\sqrt{s} \gtrsim M_s$ and some unitarization scheme has to be introduced.

2). RS scenario [2]: One anti-de Sitter dimension with a non-factorizable "warped" geometry. The physical consequences relevant to our interests are the contributions from the KK gravitons of a TeV mass [12].

3). Veneziano amplitude: Due to the lack of a fundamental theory of quantum gravity, a reasonable parameterization to the new physics at the scale M_s perhaps is to include a sum of possible "stringy" states [13]. The Veneziano amplitude serves for this purpose and manifestly preserves unitarity [9].

There are several free parameters in these models. The choices for these parameters were selected to illustrate a variety of phenomenological features and are not inclusive. In addition to the scale of quantum gravity, M_s, the ADD scenario is subject to the number of large extra dimensions, $n_{\rm D}$, although the effect under consideration is not highly sensitive to $n_{\rm D}$ [11]. The RS model varies with the scale of the theory ($\Lambda = e^{-kR} M_{pl}$), and the first graviton resonance mass (m_{KK}) as well as higher resonances. Models calculated with Veneziano amplitudes are parameterized with two numbers a and b, which parameterize the Chan-Paton traces for string models [9], in addition to M_s.

Calculations in the ADD framework were found to violate unitarity near $1.5 M_s$. To remove this behavior, partial wave amplitudes were cutoff as they saturated the unitarity bound. Calculations were then made with all partial waves (up to saturation), as well as with 5 and 10 partial waves. This variety of choices reflects our ignorance of how nature chooses to restore unitarity above this energy scale. RS models are less likely to violate unitary as the rapid growth of amplitudes only occurs way above graviton mass resonances. We include only the first graviton resonance and our calculations show that these models do not violate unitary in the energy range considered in this paper, although they will at higher energies. The use of Veneziano amplitudes automatically respects unitarity bounds [9].

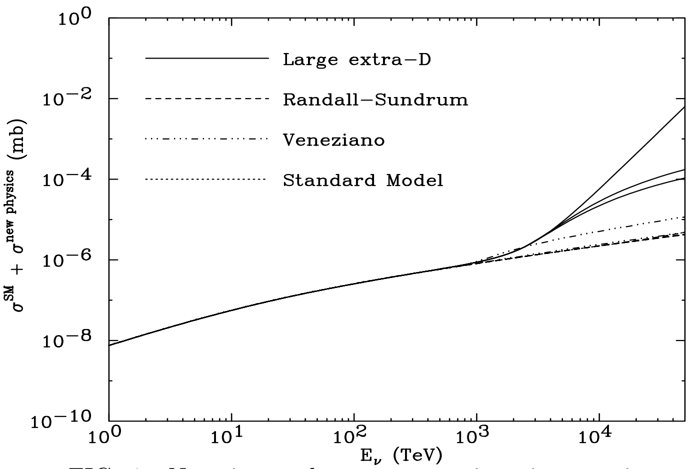

FIG. 1. Neutrino-nucleon cross sections in a variety of models compared to the standard model prediction. ADD (large extra dimension) models are for all, 10 and 5 partial waves, up to unitarity saturation (top to bottom). RS models shown are for Λ=3 TeV, m_g=500 GeV; Λ=3 TeV, m_g=1000 GeV; Λ=6 TeV, m_g=500 GeV; Λ=6 TeV and m_g=1000 GeV (top to bottom). Models using Veneziano amplitudes are for $a = b = 5$ and $a = b = 0$ (top to bottom). M_s=1 TeV for all models.

Fig.1 depicts, for the different models discussed above, the sum of the charged current neutrino nucleon cross section within the Standard Model (SM) framework and the corresponding cross section due to new physics. For the reasons explained above, we do not address the energy range above $E_\nu = 5 \times 10^4$ TeV and hence we will concentrate our investigation on the phenomenology of low-scale quantum gravity models below this maximum energy. Note that the impact of RS models is very small at these energies.

The increase in the cross section beyond the Standard Model due to TeV-scale quantum gravity starts at an energy of order $E_{\rm s}^{\nu} \sim 2 \times 10^3$ TeV. We therefore do not anticipate any observable signature of these models in neutrino telescopes below $E_{\rm s}^{\nu}$. Above $E_{\rm s}^{\nu}$ the effects of the new physics should be clearly visible for the large extra dimensions scenario [1], for which the increase in the cross section beyond the Standard Model expectations is more than 3 orders of magnitude at $E_\nu = 5 \times 10^4$ TeV. For the RS scenario [2], the influence should be almost invisible for all the graviton masses and Λ scales we have explored, since they predict a maximum increase in the cross section of only 15% at $E_\nu = 5 \times 10^4$ TeV. In between these two scenarios, the Veneziano model [9] predicts a maximum increase of order ~ 2.7 at $E_\nu = 5 \times 10^4$ TeV.

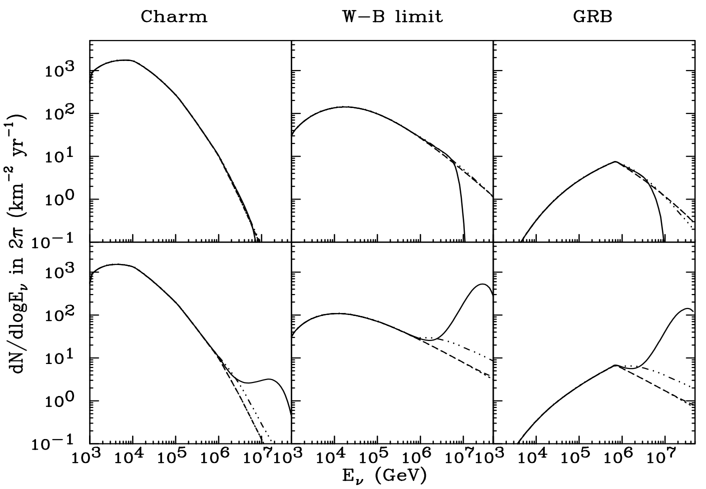

FIG. 2. Energy distribution of $\nu_\mu + \bar{\nu}_\mu$ events in Icecube. The upper panels show the upgoing neutrino events and the lower panels the downgoing events. The panels are labeled with the corresponding theoretically predicted neutrino flux that has been used to obtain the event rate (see text). In each of the panels, the solid line is the event rate when the neutrino nucleon cross section $\sigma^{\rm SM} + \sigma^{\rm ADD}$ is used; the dashed 3-dotted line corresponds to $\sigma^{\rm SM} + \sigma^{\rm Veneziano}$ with $a = b = 5$; the dashed line to $\sigma^{\rm SM} + \sigma^{\rm RS}$ with $m_{\rm g} = 500$ GeV and $\Lambda = 3$ TeV and, the dotted line corresponds to $\sigma^{\rm SM}$ alone. M_s=1 TeV for all models.

Figs.2 and 3 illustrate the qualitative behavior explained above. Fig.2 shows the energy spectrum of the expected neutrino events in Icecube for 3 different theoretical predictions of the neutrino flux. These have been chosen mostly for illustrative purposes. The panel labeled charm refers to maximal predictions of neutrinos from the decay of charmed particles produced by cosmic ray interactions in the atmosphere [16]. The W-B refers to the Waxman and Bahcall (W-B) limit on the neutrino flux from astrophysical sources that are optically thin to proton-photon and proton-proton interactions. This represents a flux of $E_\nu^2\ \Phi_\nu = 2 \times 10^{-8}$ GeV $(\text{cm}^2\ \text{s sr})^{-1}$ [19]. GRB labels the neutrino flux from Gamma Ray Bursts, energetic explosions in the Universe which occur at a rate of ~ 1000 per year. The flux accounts for fluctuations in the distance to individual GRBs and in the energy they release in the form of gamma rays [17,18]. Note that the unusual features in the angular distribution of GRB neutrinos is a result of these fluctuations [17]. Although these results can widely vary, those shown are typical.

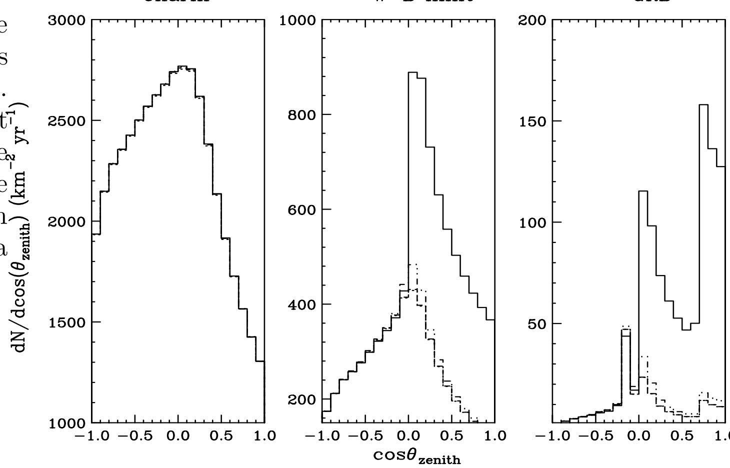

FIG. 3. $\cos(\theta_{\rm zenith})$ distribution of $\nu_\mu + \bar{\nu}_\mu$ events in Icecube. $\cos(\theta_{\rm zenith}) = -1$ corresponds to vertical upgoing neutrinos, $\cos(\theta_{\rm zenith}) = 1$ to vertical downgoing and $\cos(\theta_{\rm zenith}) = 0$ to neutrinos coming from the horizon. The panels are labeled with the corresponding theoretically predicted neutrino flux that has been used to obtain the event rate (see text). The different types of lines have the same meaning as in Fig.2.

The upper panels of Fig.2 show the upgoing neutrino events ($-1 < \cos\theta_{\rm zenith} < 0$). Fig.3 shows the zenith angle distribution of the events for the same 3 neutrino fluxes. The onset of new physics reduces the number of upgoing events due to absorption in the Earth but increases the number of events coming from the horizontal bins in zenith angle ($\cos\theta_{\rm zenith} \lesssim 0$). Both effects approximately cancel each other and at the end the rates are practically indistinguishable from the rates expected in the SM (see also Table I). A small sensitivity to new physics is then anticipated when looking for upgoing events.

The energy spectrum for downgoing events ($0 < \cos\theta_{\rm zenith} < 1$) is shown in the lower panels of Fig.2. The event rate peaks at the two neutrino energies at which the product $\Phi(E_\nu) \times \sigma_\nu(E_\nu) \times R_\mu(E_\nu)$ maximizes. The increase in the event rate due to new physics is clearly visible for the three neutrino fluxes used in the calculation whithin the large extra dimensions model. Very limited sensitivity is expected to the RS model and a mild increase in the event rate is predicted for the Veneziano scenario. The Icecube detector is sensitive to downgoing neutrinos above 1 PeV since it can determine the energy of the events and hence separate them from the background of muons produced by cosmic rays in the atmosphere, which is smaller than the neutrino induced muon flux for energies above $\sim$ 1-10 PeV [20].

The numbers in parenthesis in Table I are the rates of downgoing events above 1 PeV for the different neutrino fluxes and new physics scenarios studied in this paper. Icecube will be most sensitive to new physics when detecting neutrino fluxes that peak at energies around $E_{\rm s}^\nu$ and to the large extra dimensions scenario. The excess of events due to new physics in the charm case is very small because the flux drops quickly in the energy range where the cross section increases. The angular distribution of the downgoing events is also shown in Fig.3. Most of the excess due to new physics comes from the horizontal bins ($\cos\theta_{\text{zenith}} \gtrsim 0$), mainly because at high neutrino energy the muon range R_μ is limited by the amount of material the neutrino encounters in its way to the detector, which is only 1.8 km of ice for vertical downgoing neutrinos ($\cos\theta_{\text{zenith}} = 1$). This explains why, although the absorption of the downgoing neutrino flux is negligible, the event rate decreases from the horizon to the vertical downgoing direction.

Events/(km^2 yr) in 2π sr	Atm. charm		W-B limit		GRB	
	Up	Down	Up	Down	Up	Down
Standard Model	2422	2054 (2)	293	243 (19)	10	10 (4)
Large extra-D (all waves)	2427	2061 (5)	292	583 (358)	10	92 (86)
Large extra-D (10 waves)	2427	2059 (4)	293	357 (132)	10	33 (27)
Large extra-D (5 waves)	2427	2058 (4)	293	328 (102)	10	27 (21)
R-S, $m_g = 1$ TeV, $\Lambda = 3$ TeV	2427	2057 (2)	293	243 (19)	10	10 (4)
R-S, $m_g = 1$ TeV, $\Lambda = 6$ TeV	2427	2057 (2)	293	243 (19)	10	10 (4)
R-S, $m_g = 500$ GeV, $\Lambda = 3$ TeV	2427	2057 (2)	293	244 (19)	10	10 (4)
R-S, $m_g = 500$ GeV, $\Lambda = 6$ TeV	2427	2057 (2)	293	243 (19)	10	10 (4)
Veneziano a=b=0	2427	2057 (2)	294	244 (20)	10	10 (4)
Veneziano a=b=5	2427	2058 (3)	295	259 (35)	10	13 (8)

Table I: $\nu_\mu+\bar{\nu}_\mu$ event rate per km^2 yr in Icecube for different theoretically predicted ν fluxes and ν-nucleon cross sections. The muon energy threshold is E_μ=500 GeV and the maximum neutrino energy is E_ν (max)=5×10^7 GeV. For downgoing events the numbers in parenthesis are the event rates above E_ν =1 PeV, the energy above which the Icecube detector is able to identify downgoing neutrino induced muons from the background of atmospheric muons.

In summary, ADD models with TeV scale quantum gravity have very distinctive phenomenological features in large high energy neutrino telescopes. Models using Veneziano amplitudes can produce interesting features in some cases. RS models will be very challenging to observe by these methods. Studies of these features may allow for discovery of such models, or for stronger constraints on the scale of quantum gravity. Neutrino astrophysics can provide a method to compliment searches for TeV scale quantum gravity in collider experiments.

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